### 8.3.3 Multiple Comparisons Between Treatments

When the obtained value of $KW$ is significant, it indicates that at least one of the groups is different from at least one of the others. It does not tell the researcher which ones are different, nor does it tell the researcher how many of the groups are different from each other. What is needed is a procedure which will enable us to determine which groups are different. That is, we would like to test the hypothesis $H_0$: $\theta_u = \theta_v$ against the hypothesis $H_1$: $\theta_u \neq \theta_v$ for some groups $u$ and $v$. There is a simple procedure for determining which pairs of groups are different. We begin by obtaining the differences $|\bar{R}_u - \bar{R}_v|$ for all pairs of groups. When the sample size is large, these differences are approximately normally distributed. However, since there are a large number of differences and because the differences are not independent, the comparison procedure must be adjusted appropriately. Suppose the hypothesis of no difference among $k$ groups was tested and rejected at the $\alpha$ level of significance. We can test the significance of individual pairs of differences by using the following inequality. If

$$|\bar{R}_u - \bar{R}_v| \geq z_{\alpha/k(k-1)} \sqrt{\frac{N(N+1)}{12}\left(\frac{1}{n_u} + \frac{1}{n_v}\right)} \tag{8.6}$$

then we may reject the hypothesis $H_0$: $\theta_u = \theta_v$, and conclude that $\theta_u \neq \theta_v$. The value of $z_{\alpha/k(k-1)}$ is the abscissa value from the unit normal distribution above which lies $\alpha/k(k-1)$ percent of the distribution. The values of $z$ can be obtained from Appendix Table A.

Because it is often necessary to obtain values based upon extremely small probabilities, especially when $k$ is large, Appendix Table $A_{II}$ may be used in place of Appendix Table A. This is a table of the standard normal distribution which has been arranged so that values used in multiple comparisons may be obtained easily. The table is arranged on the basis of the number of comparisons which can be made. The tabled values are the values of $z$ associated with various values of $\alpha$. The row entries ($\#c$) are the number of comparisons. When there are $k$ groups, there are $k(k-1)/2$ comparisons possible.

**Example 8.3c** In the large-sample example in this section, we rejected $H_0$ and concluded that the medians were not equal. Since there are $k = 3$ groups, there are $3(3-1)/2 = 3$ comparisons possible. If we take the differences between the average rankings, we have

$$|\bar{R}_1 - \bar{R}_2| = |4.17 - 10.83| = 6.66$$

$$|\bar{R}_1 - \bar{R}_3| = |4.17 - 13.50| = 9.33$$

$$|\bar{R}_2 - \bar{R}_3| = |10.83 - 13.50| = 2.67$$

To find which of these comparisons is significant, we can apply the multiple-comparison test described in this section. It is necessary to find the critical value of $z$. Since we chose $\alpha = .05$ in the original analysis, the same level shall be used here, and since the number of comparisons is $\#c = k(k-1)/2 = 3(3-1)/2 = 3$, we may find the critical value of $z$ from Appendix Table $A_{II}$; that value is $z = 2.394$. [This is the same value we would obtain from Appendix Table A: $z_{\alpha/k(k-1)} = z_{.05/3(3-1)} = z_{.0083} \approx 2.39$.] The critical difference is then found by using Eq. (8.6):

$$z_{\alpha/k(k-1)}\sqrt{\frac{N(N+1)}{12}\left(\frac{1}{n_u}+\frac{1}{n_v}\right)} = 2.394\sqrt{\frac{18(18+1)}{12}\left(\frac{1}{6}+\frac{1}{6}\right)} \tag{8.6}$$

$$= 2.394\sqrt{9.5}$$

$$= 7.38$$

Since only the difference between groups 1 and 3 (the irrelevant cue first versus both cues) exceeds the critical value 7.38, only that comparison was significant and it may be concluded that these medians are different.

It should be carefully noted that, in the application of Eq. (8.6) to the multiple comparisons in the above example, only a single critical difference was calculated. This was possible because each of the $k$ groups was equal in size. Had the sample sizes been unequal, each of the observed differences would have to be compared against different critical differences.

**COMPARISONS OF TREATMENTS VERSUS CONTROL.** Sometimes a researcher includes a control group or standard group as one of the $k$ groups. An example would be when a researcher wishes to assess the effects of various drugs on behavior. Although a major interest may be on whether the groups are different on the measured variable, the primary concern may be whether there is a difference between the behavior under administration of *any* of the drugs and behavior when no drug (or a placebo) is administered. In this case the researcher would still apply the Kruskal-Wallis one-way analysis of variance by ranks if the assumptions for its use were appropriate. However, if $H_0$ is rejected, the researcher is concerned about whether any of the drug groups differ from the control group. That is, if $\theta_c$ is the median for the control group, and $\theta_u$ is the median for the $u$th group, the researcher would like to test $H_0$: $\theta_c = \theta_u$ against $H_1$: $\theta_c \neq \theta_u$ (or perhaps $H_1$: $\theta_c > \theta_u$). Since we are not interested in comparing all groups, the multiple-comparison method given by Eq. (8.6) must be adjusted to account for the smaller number of comparisons. When there are $k$ groups in the overall test, there will be $k - 1$ comparisons with a control; thus $\#c = k - 1$. The appropriate relations for the multiple comparisons in this case are the following:

To test $H_1$: $\theta_c \neq \theta_u$,

$$|\bar{R}_c - \bar{R}_u| \geq z_{\alpha/2(k-1)}\sqrt{\frac{N(N+1)}{12}\left(\frac{1}{n_c}+\frac{1}{n_u}\right)} \tag{8.7}$$

To test $H_1$: $\theta_c > \theta_u$,

$$\bar{R}_c - \bar{R}_u > z_{\alpha/(k-1)}\sqrt{\frac{N(N+1)}{12}\left(\frac{1}{n_c}+\frac{1}{n_u}\right)} \tag{8.8}$$